Electrical Technology

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NETWORK DEFINITIONS

LINEAR SYSTEM : A system(network) is linear if (a) the principle of SUPERPOSITION and(b) the principle of PROPORTIONALITY hold.

SUPERPOSITION PRINCIPLE : If for a given network,{ e_1 (t), r_1(t) } and {e_2(t),r_2 (t) } are excitation-response pairs, then if the excitation were e(t) = e_1 (t) +e_2 (t) , the response would be r(t) = r_1 (t) + r_2 (t) .

PROPORTIONALITY PRINCIPLE If for a given network, e_1 (t), r_1(t) are excitation-response pairs, then if the excitation were c_1 e_1 (t) , then response would be $c_1 r_1$ (t).The constant of proportionality is c_1 is preserved by the linear network. In a system if

$$c_1 e_1 (t) + c_2 e_2 (t) \quad \text{gives rise to} \quad c_1 r_1 (t) + c_2 r_2 (t) .$$

Then the system is a linear system

PASSIVE SYSTEM A linear network is passive if (a) the energy delivered to the network is non-negative for any arbitrary excitation , and (b) if no voltages or currents appear between any two terminals before an excitation is applied.

NETWORK ANALYSIS (CONTD)

- **RECIPROCAL** A network is said to be **reciprocal** if when the points of excitation and measurement of response are interchanged , the relationship between excitation and response remains the same. This must be true for any choice of points of excitation & response.
- **CAUSAL** A system is causal if its response is non-anticipatory, i.e.,if

$$e(t) = 0 \qquad t < T$$

then $\quad r(t) = 0 \qquad t < T$

in other words , a system is causal if before an excitation is applied at $t = T$, the response is zero for $-\infty < t < T$.

TIME INVARIANT A system is time invariant if $e(t) \rightarrow r(t)$ implies that $e(t \pm T) \rightarrow r(t \pm T)$. From this property we can show that if e(t) at the input gives rise to r(t) at the output then if the input were $e`(t)$ i.e. the derivative of e(t) , the response would be $r`(t)$.

This idea is applicable to higher derivatives as well as for the integrals of e(t) and r(t) .

IDEAL MODELS AND ELEMENTS

- **AMPLIFIER :** An amplifier scales up the magnitude of the input , i.e., $r(t) = K\, e(t)$, where K is a constant.
- **DIFFERENTIATOR :** The input signal is differentiated and possibly scaled up or down.
- **INTEGRATOR** The output is the integral of the input.
- **TIME DELAYER** The output is delayed by an amount T , but retains the same wave shape as the input
- **IDEAL ELEMENTS/ ENERGY SOURCES.** Resistor R given in ohms, the Capacitor C ,given in farads , and the Inductor L , expressed in Henrys are the main elements used in networks in addition to current or voltage energy sources.These energy sources could be independent or dependent . The dependent sources could be either voltage controlled or current controlled.

GENERAL CHARACTERISTICS OF SIGNALS

- **PERIODIC OR APERIODIC SIGNAL** A signal is said to be periodic if it can be described by the equation as $f(t) = f(t \pm kT)$ k = 0, 1 , 2 , ... Where T is the period of the signal . The sine wave , sin t , is periodic with period T = 2π . Another example of periodic signal is the Square Wave .
- Signals like pulses (rectangular , triangular) are not periodic because the pulse patterns do not repeat after certain finite interval T .
- For discrete time signal x(n) , the condition of periodicity can be written as
- $x(n + N_o) = x(n)$ where N_o is sampling period or no of samples after which signals repeat itself.
- A signal which does not repeat itself after a fixed time period or does not repeat at all is aperiodic or non periodic signal.
- **PERIODICITY OF SUM OF TWO PERIODIC SIGNALS**

The sum of two periodic continuous time signals will be periodic if the ratio of their fundamental period is rational.

(A discrete time signal is is periodic only if frequency f_o is rational i.e. in form of two integers)

GENERAL DESCRIPTION OF SIGNALS

- **Time Constant** It refers only to exponential waveforms .It is a useful measure of the decay of an exponential . Consider an exponential waveform described by

  $r(t) = K\, e^{-t/T} u(t)$ from the plot of this function we see that when $t = T$,

  $r(t) = 0.37\, r(0)$

  also $r(4T) = 0.02\, r(0)$. This shows that the larger the time constant ,the longer it requires for the waveform to reach 37 % of its peak value. In circuit analysis ,the common time constants are the factors RC and RL .
- RMS Value , D-C Value , Duty Cycle and Crest Factor are other terms which describe only Periodic Waveforms.

r m s value

- The r.m.s value of an alternating current is given by
- **That steady (d.c) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time**.
- It is also known as the effective or virtual value of the alternating current. The term EFFECTIVE value is used more extensively.S

<u>GENERAL DESCRIPTION OF SIGNALS (CONTD)</u>

- **<u>RMS Value</u>** The rms or root mean square value of a periodic waveform e (t) is defined

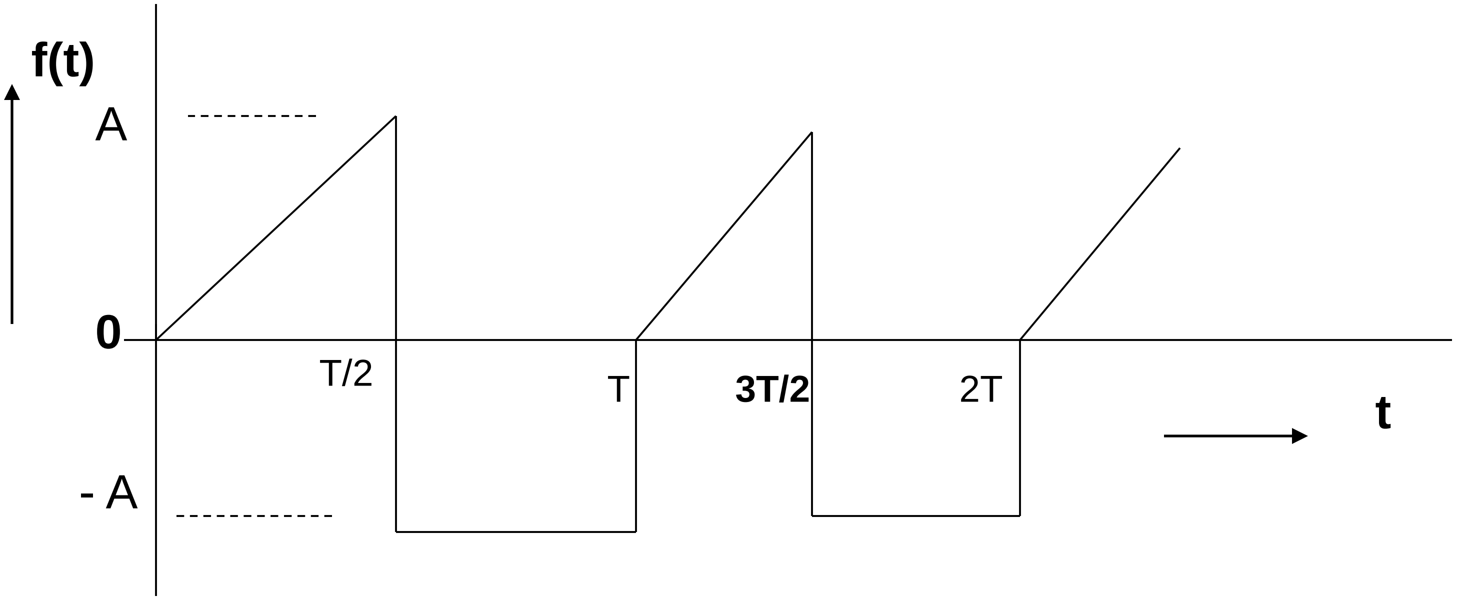
$$e_{rms} = \left[1/T \int_0^T e^2(t)\, dt \right]^{1/2}$$ where T is the period . If the

waveform is not periodic ,the term rms does not apply .

Show that for a periodic waveform which is triangular in period 0 to T/2 with amplitude increasing from 0 to A v and rectangular in next half period T/2 to T with amplitude - A v during this half period .The rms value works out to be

$\sqrt{2/3}$ A v

Ex : find rms value of the waveform shown below



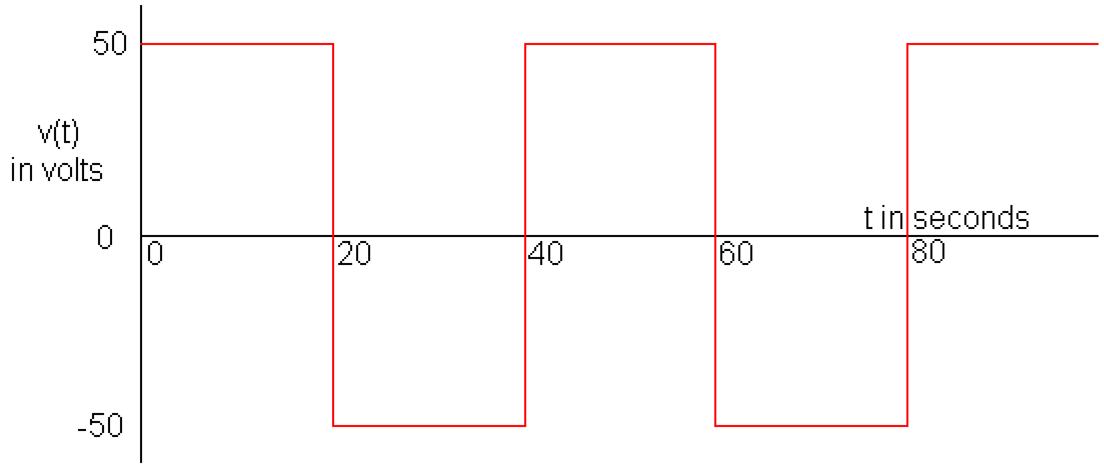
Solution

- $$E_{rms} = \left[1/T \left\{ \int_0^{T/2} (2A\,t/T)^2\,dt + \int_{T/2}^{T} A^2\,dt \right\}\right]^{1/2}$$

$$= \left[1/T \left\{ (4A^2/T^2)\, t^3/3 \Big|_0^{T/2} + A^2 t \Big|_{T/2}^{T} \right\} \right]^{1/2}$$

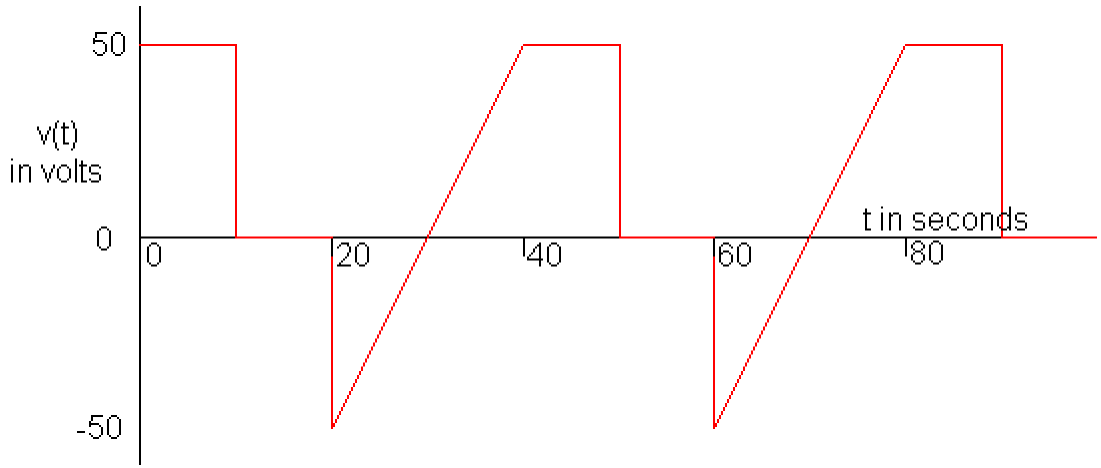
$$= A\sqrt{2/3}$$

- Find rms value of the waveform shown below



$$V_{rms} = \{[50^2 + (-50)^2]/2\}^{1/2} = \{[2500 + 2500]/2\}^{1/2} = \{2500\}^{1/2} = 50\ V$$

- Find rms value of the waveform shown below



Solution

$$V_{rms} = \sqrt{\frac{1}{40}\left[\int_{0}^{t_1} v^2\,dt + \int_{t_1}^{t_2} v^2\,dt + \int_{t_2}^{40} v^2\,dt\right]}$$

$$V_{rms} = \sqrt{\frac{1}{40}\left[\int_{0}^{10} v^2\,dt + \int_{10}^{20} v^2\,dt + \int_{20}^{40} v^2\,dt\right]}$$

$$V_{rms} = \sqrt{\frac{1}{40}\left[\int_{0}^{10} 2500\,dt + \int_{10}^{20} 0\,dt + \int_{20}^{40} (5t - 150)^2\,dt\right]}$$

Solution (contd)

$$V_{rms} = \sqrt{\frac{1}{40}\left[\ 25000 \ + \ 0 \ + 16667\ \right]}$$

$$V_{rms} = \sqrt{41667/40}$$

$$V_{rms} = 32.27 \text{ V}$$

This rounds off to $V_{rms} = 32$ V

Average or d.c value

- The average value of an alternating current is expressed as
- **That steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.**
- **In the case of a symmetrical alternating current (i.e one whose two half cycles are similar , whether sinusoidal or non sinusoidal), the average value over a complete cycle is zero. So, in their case , the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle. But** in the case of an unsymmetrical ac (like half-wave rectified current) the average value must always be taken over the whole cycle.

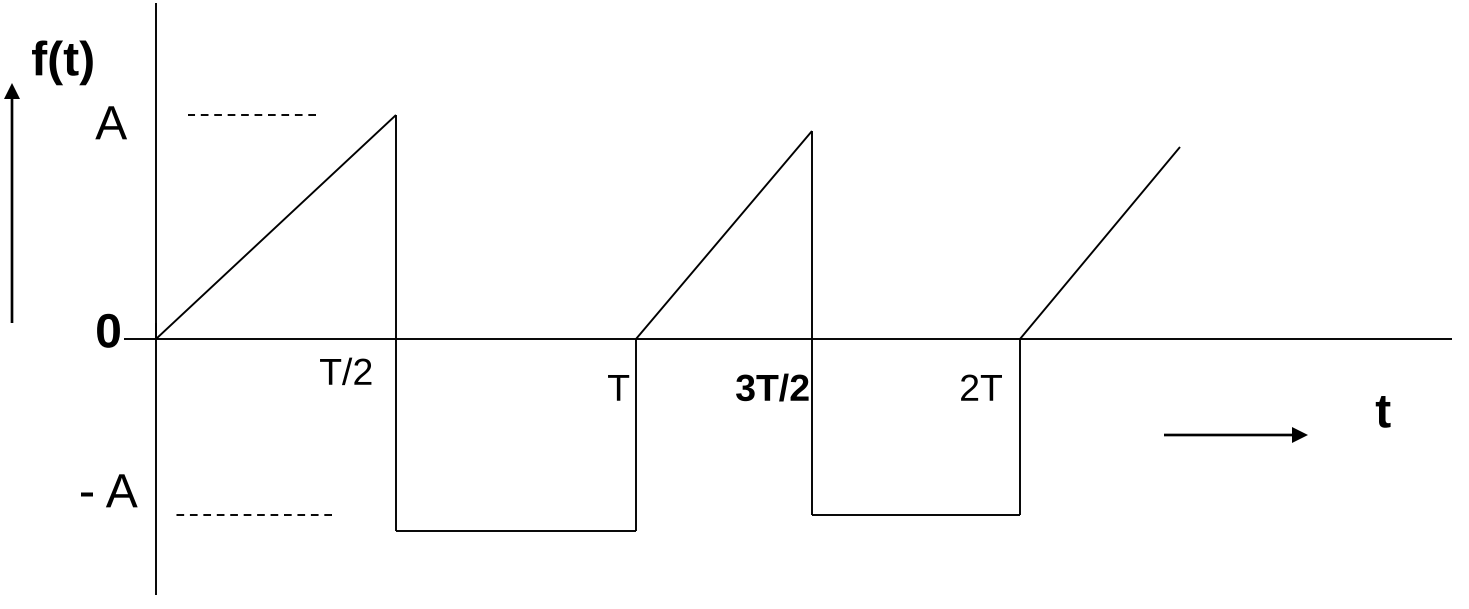
GENERAL DESCRIPTION OF SIGNALS (CONTD)

- **D-C Value** The d-c value (or average vaue) of a waveform has meaning only when the waveform is periodic . It is the average value of the waveform over one period

$$e_{d-c} = 1/T \int_0^T e(t)\,dt$$ where T is the period .

The d-c value of the wave form described earlier works out to be - A/4 v.

Ex : find average value of the waveform shown below

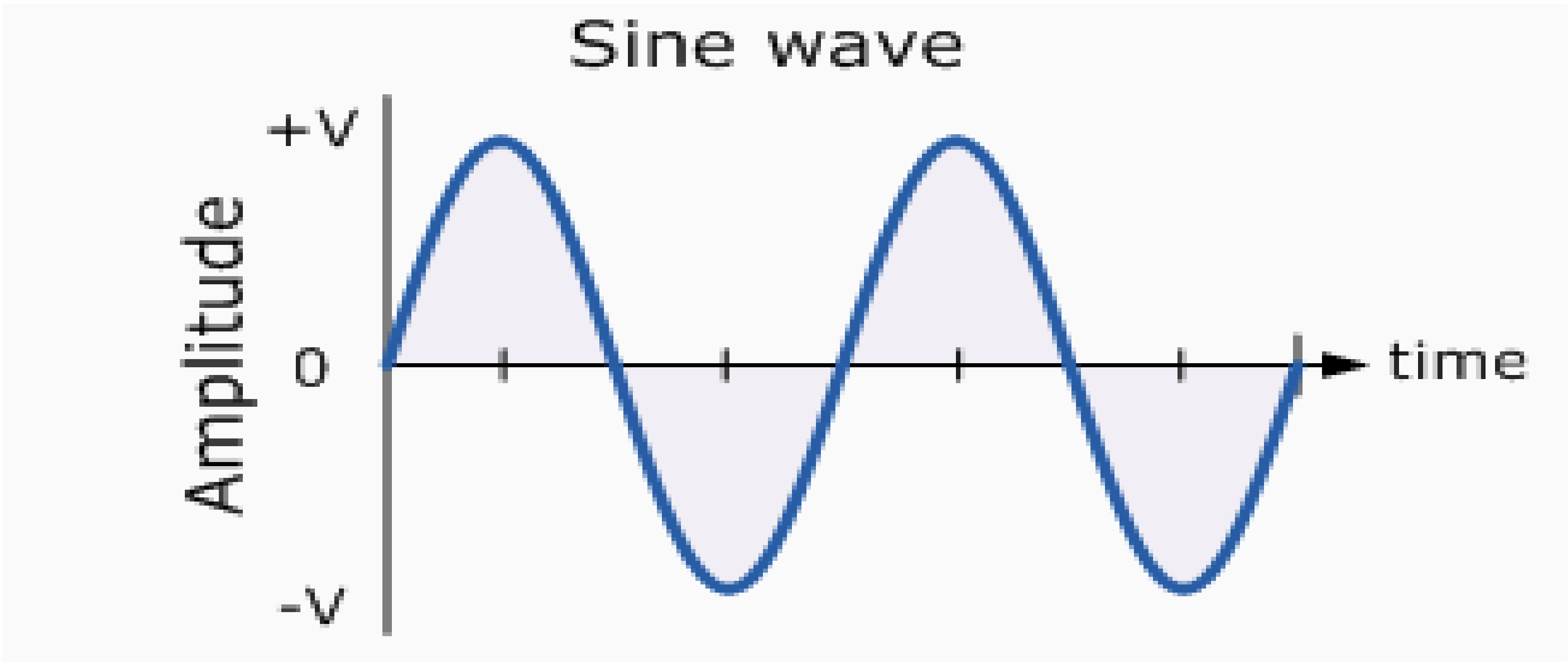


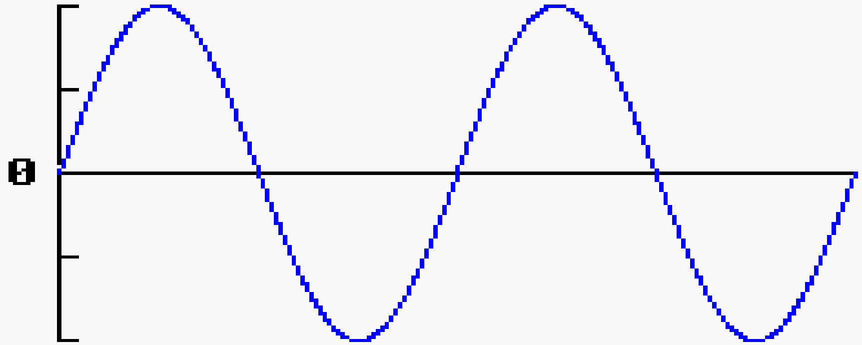
Solution

- $$E_{dc} = 1/T \int_0^T e(t)\, dt$$

$$= 1/T \left[\int_0^{T/2} (2A/T)\, t\, dt + \int_{T/2}^{T} (-A)\, dt \right.$$

$$= -A/4$$


Sine wave
+V
Amplitude
0
-V
time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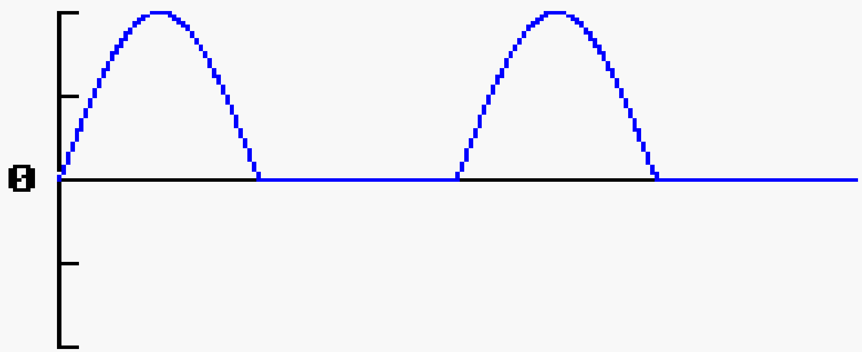
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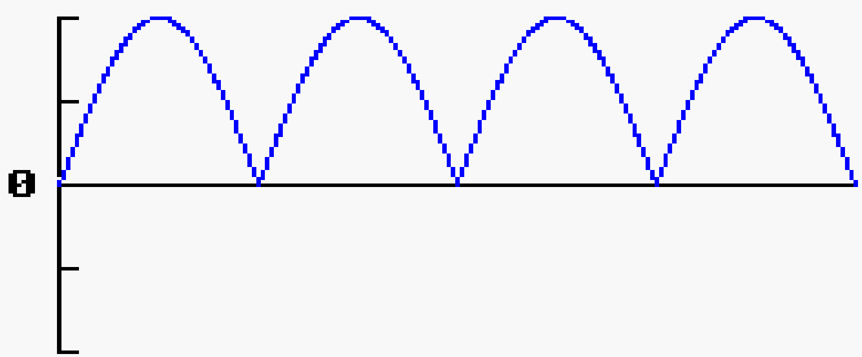
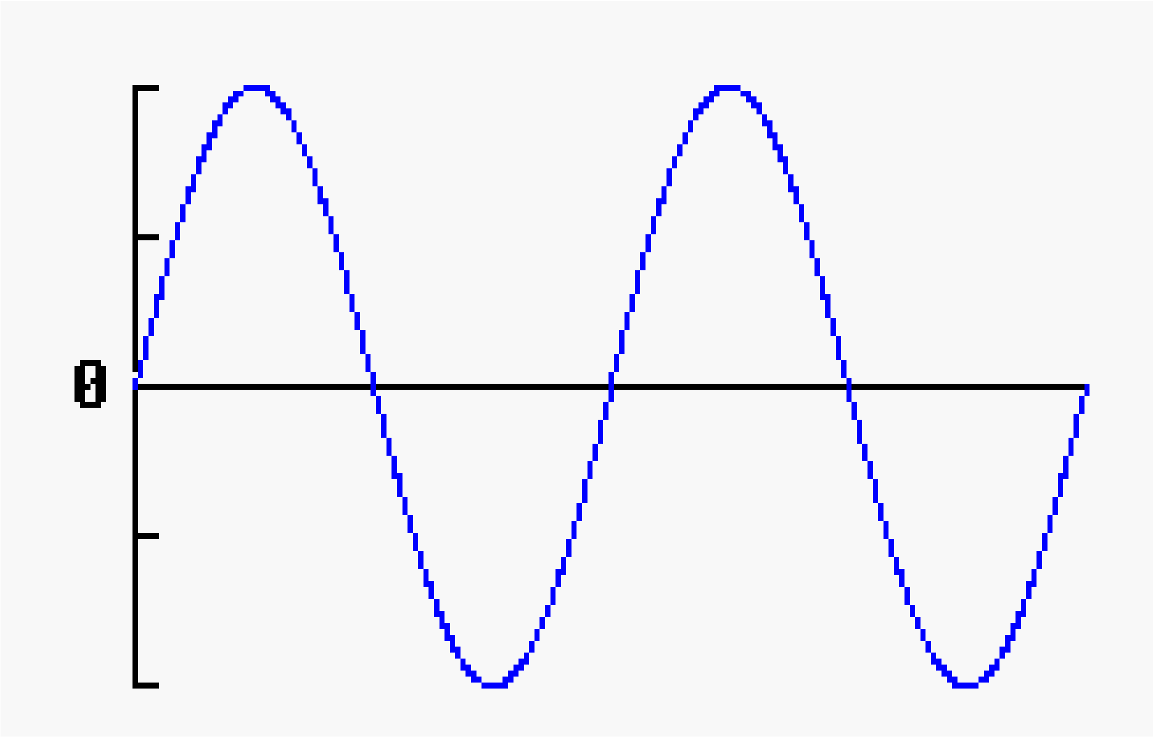

0

rms & average values of sine waves

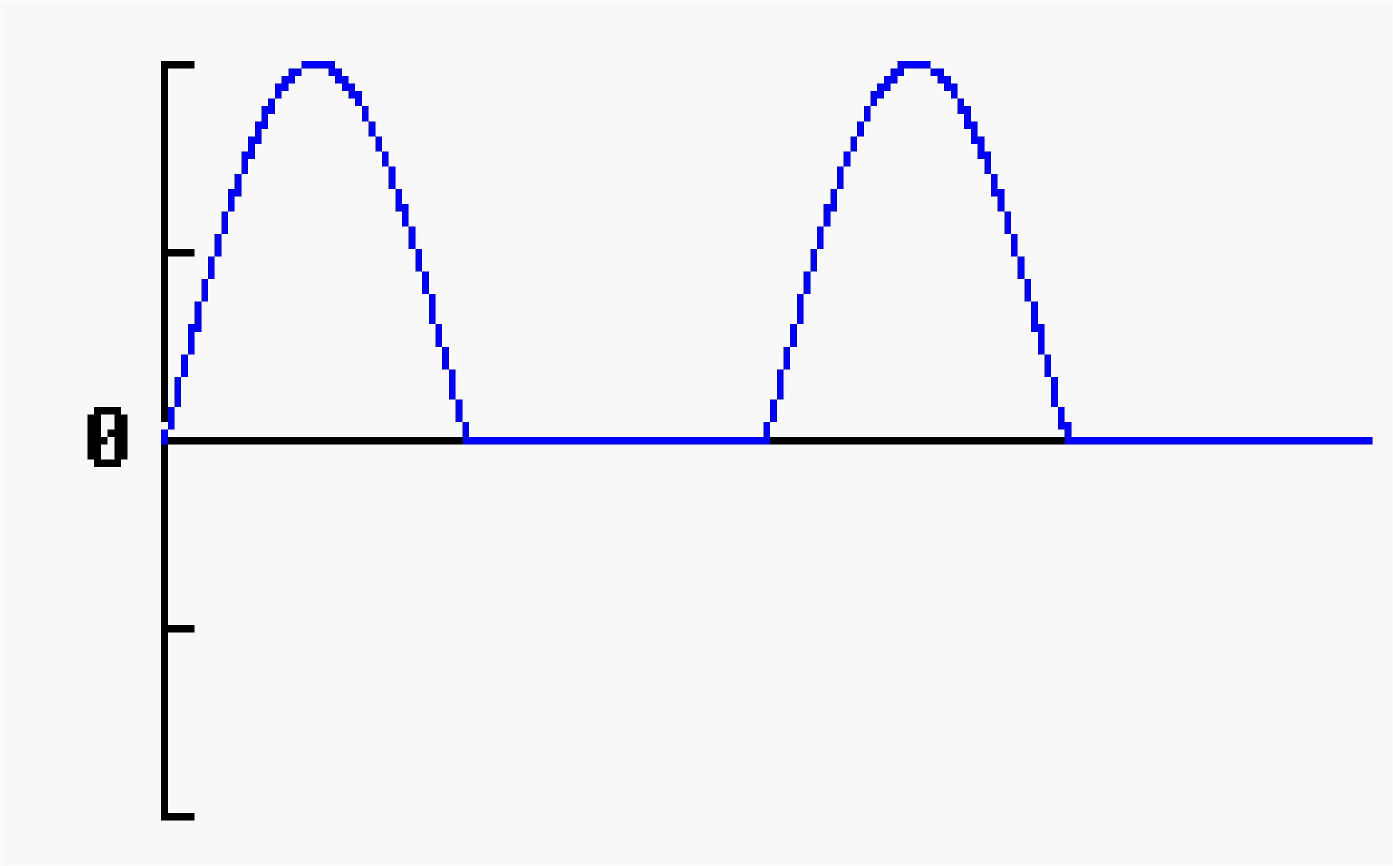
- $V = V_m \text{ Sin } wt = V_m \text{ Sin } \theta$



rms value = $V_m / \sqrt{2}$ average value = 0

rms & average values of sine waves

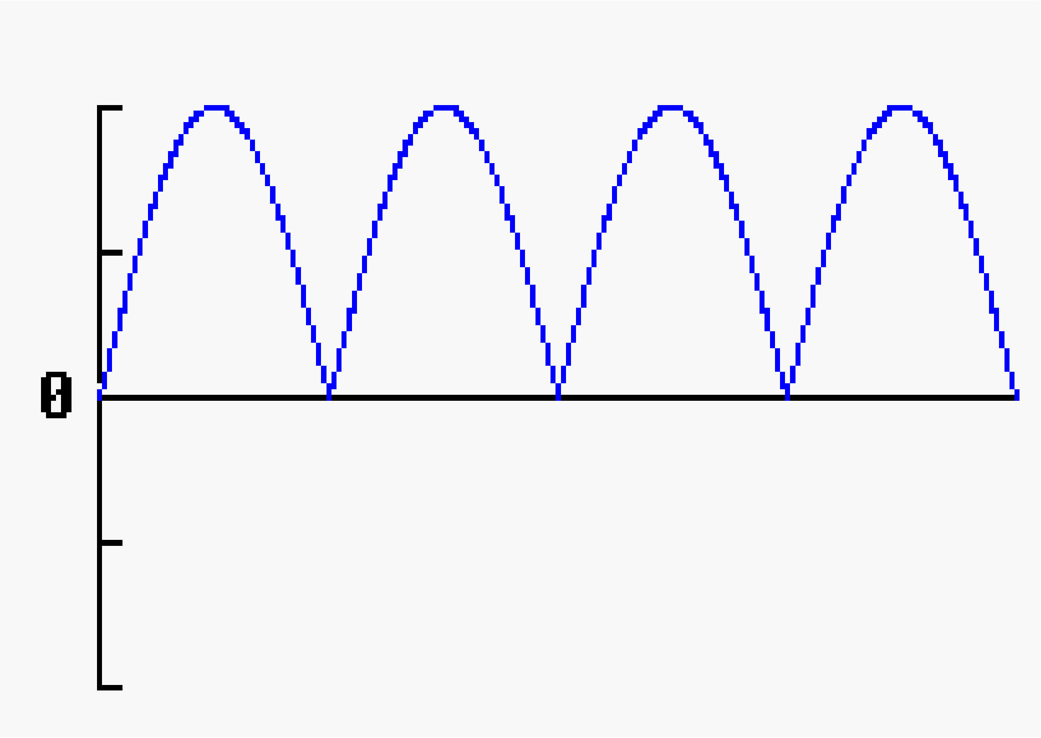
- $V = V_m \text{ Sin wt} = V_m \text{ Sin } \theta$



rms value = V_m / 2 **average value = V_m / π**

rms & average values of sine waves

- $V = V_m \sin wt = V_m \sin \theta$



rms value = $V_m / \sqrt{2}$ **average value = $2 V_m / \pi$**

GENERAL DESCRIPTION OF SIGNALS (CONTD)

- **DUTY CYCLE** The term Duty Cycle , D , is defined as the ratio of the time duration of the POSITIVE CYCLE t_o of a periodic waveform to the period ,T , that is

$$D = \frac{t_o}{T}$$

CREST FACTOR Crest factor is defined as the ratio of the peak voltage of the periodic waveform to the rms value (with the d-c component removed).

FORM FACTOR form factor is defined as the ratio of r.m.s value to average value of the waveform.